UPPER MANTLE CONDUCTIVITY DETERMINED FROM THE SOLAR QUIET DAY IONOSPHERIC CURRENTS IN THE DIP EQUATORIAL LATITUDES OF WEST AFRICA

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Abstract

The magnetic data obtained from a chain of ten magnetotelluric stations installed in the African sector during the international equatorial electrojet year (IEEY) was used to establish the 1993 quiet day current system (Sq) for West Africa and to determine the Earth’s upper mantle electrical conductivity in the region. A spherical harmonic analysis (SHA) was applied in the separation of the internal and external field/current contribution to the Sq variations, while a special transfer function was used to compute the conductivity – depth values from the paired external and internal coefficient of the SHA. The variation in the currents is seen to be a dawn to dusk phenomenon with the variation in the external currents different from that of the internal currents both in amplitude and in phase. The seasonal variation in the external current maximizes during the March equinox and minimizes in the December solstice. The conductivity had a downward increase with a high conductivity region spotted between 100 km and 205 km, which is seen to correspond to the seismic low velocity region. The conductivity at the upper mantle is seen to be 1.05 times higher than that obtained both in the Asian (Himalayan) and Australian regions.

1. Introduction

A varying electric current flowing in the earth’s upper atmosphere (the ionosphere) induces a corresponding electric current to flow in the solid earth. The magnetic observatories on quiet days measure a composite mixture of both the source (external) currents and induced (internal) currents. Separating these currents into their individual parts using Spherical Harmonic Analysis (SHA) or other integral methods, the amplitudes and phase relationships were shown to be useful in determining the conductivity of the deep earth [1]. The depth of penetration of the induced current to the deep earth depends on the period of variation of the source current and the distribution of electrically conducting materials in the region of the earth begin investigated.

The authors of [2, 3] applied the SHA method to geomagnetic field data obtained on quiet days to determine the electrical conductivity of the upper mantle in the Asian (Himalayan) region and Australian region. The authors of [4] tried to determine the conductivity of the upper mantle for the seven continents of the world by applying the SHA method to geomagnetic field data obtained on quiet days. In the African continent, they used data obtained from the Southern hemispheric region.
The aim of this work is to separate the quiet-day field variations obtained in the West African region into their external and internal field contributions, to establish the external $S_q$ current system for West Africa, and then to use the paired external and internal coefficient of the SHA to determine the earth’s upper mantle conductivity for the region.

The data employed in the analysis consist of hourly mean values of geomagnetic field (H, D, and Z) elements obtained on solar quiet days in 1993. The data were obtained from a record of a chain of ten geomagnetic stations installed during the French participation in the International Equatorial Electrojet Year (IEEY) in Africa. The ten stations involved are Tombouctou (TOM), Mopti (MOP), San (SAN), Koutiala (KOU), Sikasso (SIK), Nielle (NIE), Korhogo (KOR), Katiola (KAT), Tiebissou (TIE), and Lamto (LAM). These stations are located between Ivory Coast in the South and Mali in the North. Figure 1 is a graphical presentation of the ten West African stations and three permanent observatories in the region.

**Fig. 1.** The geographic location of the stations of the IEEY electromagnetic profile (*), three permanent African magnetic observatories (●). The Z = 0 line corresponds to the 1993 IGRF dip equator [5].

### 2. Method of analysis

The analytical method employed in this work involves the SHA devised by [6] in solving the magnetic potential function $V$. It was the author of [6] who showed that the potential has two parts: the external (source) and internal (induced) parts of the potential function. The magnetic potential $V$ at geocentric distance $r$ can now be expressed as

$$V = C + R\sum_n \sum_m \left[ V^{me}_n + V^{mi}_n \right],$$

where $C$, $R$, $V^{me}_n$, and $V^{mi}_n$ denotes the constant of integration, the radius of the earth, the external and internal potential, respectively.

Since we have another evidence that the dynamo current source is in the E-region ionosphere near 100-km altitude, then following [7] we can now write the external current function $J_e(\phi)$ in amperes for an hour of the day as

$$J_e(\phi) = \sum_{m=1}^{4} \sum_{n=1}^{12} \left( \frac{5R}{2\pi} \right) \frac{2n+1}{n+1} \left( a^{me}_n \cos(m\phi) + b^{me}_n \sin(m\phi) \right) p^n_m(\theta);$$

the internal current function $J_i(\phi)$, in amperes for an hour of the day

$$J_i(\phi) = \sum_{m=1}^{4} \sum_{n=1}^{12} \left( \frac{5R}{2\pi} \right) \frac{2n+1}{n} \left( a^{mi}_n \cos(m\phi) + b^{mi}_n \sin(m\phi) \right) p^n_m(\theta).$$

Here $a^{me}_n, a^{mi}_n, b^{me}_n$ and $b^{mi}_n$ are the Legendre polynomial coefficients, where $e$ and $i$ represent the external and internal values, respectively; $p^n_m$ are the Legendre polynomials and are functions of colatitude $\theta$ only. The integers $n$ and $m$ are called degree and order respectively; $n$ has a value of 1 or greater, and $m$ is always less than or equal to $n$.

The transfer equations necessary for obtaining conductivity versus depth profile from the separated external and internal SHA is given in [8] as
\[ C_n^m = z - ip; \]  
\[ Z = \frac{R}{n(n+1)} \left\{ \frac{a_n^m [na_n^m - (n+1)a_n^m] + b_n^m [nb_n^m - (n+1)b_n^m]}{(a_n^m)^2 + (b_n^m)^2} \right\} \]  
\[ P = \frac{R}{n(n+1)} \left\{ \frac{a_n^m [nb_n^m - (n+1)b_n^m] - b_n^m [na_n^m - (n+1)a_n^m]}{(a_n^m)^2 + (b_n^m)^2} \right\}, \]  
where \( R, Z, \) and \( P \) are given in kilometers. The coefficient sums are given by 
\[ a_n^m = [a_n^{mc} + a_n^{mi}], \quad b_n^m = [b_n^{mc} + b_n^{mi}]. \]

For each \( n, m \) set of coefficient, the depth (in km) to the uniform substitute layer is given by 
\[ d_n^m = Z - P, \]  
and the conductivity \( \sigma \) (in S/m) is 
\[ \sigma_n^m = \frac{5.4 \times 10^4}{m(\varphi p)^{1/2}}. \]  
The data processing followed the steps shown below in Fig. 2.

1. Select the five Internationally Quiets days of each month for the year 1993.
2. Evaluate monthly mean values of H, D, and Z field elements for the five selected quiet days.
3. Carryout a Fourier analysis (4 harmonics) of each component after trend and base line removal (with daily mean as the base line).
4. Create file of Fourier analysis coefficients for recreation of quiet day analysis record for any day/month for any station.
5. Carryout spherical harmonic analysis with degree \( n = 12 \), order \( m = 4 \) obtaining the Legendre polynomial external and internal coefficients.
6. Determine conductivity versus depth values for each day/month and for each pair of \( m \) and \( n \).
7. External, Internal and total field recreation for any day/month for at the given latitude and longitude.
8. Regression fits for depth versus conductivity profile.
9. External, Internal and total current for any day/month at the given latitude and longitude.

Fig. 2. Data Processing Routine.
Results and discussion

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Fig. 3. Separated External, Internal, and Total currents for Lamto (LAM) and Tombouctou (Tom) for the month of March.

Figure 3 illustrates the separated currents: external, internal, and total (external + internal) currents in the West African region for the month of March in 1993. The station Lamto (LAM) is used to represent the stations located northern side of the dip equator, while Tombouctou (TOM) represents the stations located southern side of the dip equator. The variation in both the external and internal currents occurred in all hours of the day from dawn to dusk. The observed variation in internal currents is seen to be different from the external currents both in amplitude and in phase. These differences were observed in the phase and amplitude is a function of the Earth's conductivity in [4]. The external current pattern in the northern stations is seen to be exactly opposite to that in the southern stations. The midday increase at the northern most stations is matched by a midday decrease in the southern most stations. The external current range provided a simple way to view a full year's change in the external Sq current system. The maximum range was found in March in the station Katiola with a value of 34.3 x 10^6 A, and the minimum range was found in December in the station San with a value of 1.7 x 10^6 A. The yearly averages of March, June, and December were used to represent the equinoctial, summer, and winter solsticial variations of these external currents. We found an equinoctial maximum (March equinox) with a value of 16.6 x 10^6 A and a solsticial minimum in December (Winter solstice) with a value of 7.76 x 10^6 A.

One of the critical features affecting the Sq pattern is solar ionization, on equinox when the sun is overhead at the equator it is expected that the source currents will be on the increase since the stations are located at the equator. This result is in line with that obtained in [9], where it is found that the temperature in the upper atmosphere estimated from satellite drag on geomagnetically quiet days is higher during equinox than during summer.

Figure 4 displays the Electrical conductivity-depth profile of the upper mantle and transition zone based on the West African solar quiet day variation. The small squares represent the conductivity-depth computation results, while the solid line is the regression fitted values. The plot gave a scatter which could result from error in the SHA fitting, variability of source
The profile depicts a downward increase in conductivity from a depth of about 100 km to about 1200 km with a sharp increase observed from 0.037 S/m at a depth of 100 km to about 0.09 S/m at 205 km.

From the shape of the conductivity–depth profile, the upper mantle can be viewed as a stack of inhomogeneous layers with a downward increase in conductivity, which agrees with the global models of a steep rise in conductivity from 300 km - 700 km [2, 4, 10-14]. The rise in conductivity from about 0.037 S/m in 100 km to 0.09 S/m in 205 km is seen to correspond to the global seismic low velocity region, the asthenosphere [15, 16]. The global mantle seismic discontinuity at around 400 km and 670 km were not evident in the profile.

Comparing our work with that obtained in the Australian and Asian regions, we found our upper mantle conductivity values to be 1.05 times higher than that obtained both in the Asian (Himalayan) region by [2] and in the Australian region by [3]. In Africa (southern hemisphere, the authors of [4] found high conductivity values between about 150 km and 350 km and a general increase afterwards, which is in agreement with our result even though our stations are located in the Northern hemisphere.

The analysis of the upper oceanic asthenosphere and the inference of a high conductivity zone at about 200 km depth of [17] equally agree with our result. We could see correspondence existing between the high conductivity zone and the low velocity zone, this correspondence is in agreement with the global results of [18].

Having compared our results with data obtained in other regions of the world, we therefore infer from our work that below the 400-km depth, the upper mantle under West Africa is highly conductive.

Conclusions

The application of the solar quiet day ionosphere current has enabled us to establish the West African external Sq current system and also to determine the conductivity depth structure of the upper mantle in the West African region.

The following deductions can be made from the results:
(i) The variation in the currents is seen to be a dawn to dusk phenomenon;
(ii) The source currents varied from the induced currents both in amplitude and in phase;
(iii) The equinoctial maximum is observed in external current intensity which occurred during the March Equinox;
(iv) A downward increase in conductivity is steeper at the upper mantle than at the transition zone;
(v) A high conductivity region spotted between 100 km and 205 km corresponds to the low velocity region.

Therefore, we conclude from this work that below the 400-km depth the upper mantle under West Africa is highly conductive. We suggest that further work should be carried out in other parts of West Africa as this will help to throw more light on the upper mantle conductivity in the region.
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References