RADIATION AND ROTATION EFFECTS ON AN ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND UNIFORM MASS

A. R. Vijayalakshmi\textsuperscript{1} and A. Florence Kamalam\textsuperscript{2}

\textsuperscript{1}Department of Applied Mathematics, Sri Venkateswara College of Engineering, Sriperumbudur - 602 105, INDIA, E-mail: avijaya@svce.ac.in
\textsuperscript{2}Panimalar Engineering College, Nazarethpet, INDIA

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Abstract

An analysis of effects of thermal radiation and rotation on an accelerated vertical plate with variable temperature and uniform mass is presented. An exact solution is obtained for the axial and transverse components of the velocity by defining a complex velocity. The effects of velocity, temperature and concentration for different parameters, such as radiation parameter, rotation parameter, Schmidt number, thermal Grashof number, mass Grashof number, Prandtl number, and time on the plate, are discussed.

1. Introduction

The effect of radiation is quite significant at high temperatures. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines, and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of these engineering applications. The effect of coriolis force is widely applied in science and technology.

Arpaci [2] studied the interaction between thermal radiation and laminar convection of a heated vertical plate in a stagnant radiating gas. England and Emery [3] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Mass transfer effects on a free convection flow past a semi-infinite plate was presented by Gebhart and Pera [4]. Bestman and Adjepong [5] studied the magnetohydrodynamic free convection flow, with radiative heat transfer, past an infinite moving plate in rotating incompressible, viscous, and optically transparent medium. Das et al. [6] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. Raptis and Perdikis [7] considered the effects of thermal radiation and free convection flow past a moving vertical plate. Again, Raptis and Perdikis [8] investigated free convection and mass transfer effects on optically thin gray gas past an infinite moving vertical plate. The governing equations were solved analytically. Gupta et al. [9] have analyzed free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid. Kafousias and Raptis [10] extended this problem to include mass transfer effects subjected to variable suction or injection.

Singh [11] studied the effects of coriolis as well as magnetic force on the flow field of an electrically conducting fluid past an impulsively started infinite vertical plate. Chandran et al. [12] had studied the Natural convection near a vertical plate with ramped wall temperature. Thermal stratification effects on unsteady natural convection flow past an accelerated vertical plate was studied by Deka and Neog [13]. Kesavaiah et al. [14] studied the thermal radiation and heat generation effects on unsteady flow of a viscous incompressible fluid past an impulsively
started infinite vertical plate with variable temperature and uniform mass diffusion.

However, heat and mass transfer effects on an accelerated vertical plate in a rotating fluid in the presence of thermal radiation has not been studied in the literature. It is proposed to study thermal radiation effects on the flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion, in a rotating fluid. The dimensionless governing equations are solved by the Laplace transform technique.

2. Mathematical model

Consider the three dimensional flow of a viscous incompressible fluid induced by uniformly accelerated motion of an infinite vertical plate with variable temperature and uniform mass diffusion in a rotating fluid [4, 5]. On this plate, the $x'$-axis is taken along the plate in the vertically upward direction and the $y'$-axis is taken normal to $x'$-axis in the plane of the plate and $z'$-axis is normal to it. Both the fluid and the plate are in a state of rigid rotation with uniform angular velocity $\Omega'$ about the $z'$-axis. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Initially, the plate and fluid are at rest with temperature $T_{\infty}'$ and concentration $C_{\infty}'$ everywhere. At time $t'$ > 0, the plate starts moving with a velocity $ct'$ in its own plane in the vertical direction against gravitational field in the presence of thermal radiation. At the same time the plate temperature is raised to $T_w'$ and the concentration to $C_w'$, which are thereafter maintained constant. Since the plate occupying the plane $z' = 0$ is of infinite extent, all the physical quantities depend only on $z'$ and $t'$. Then by Boussinesq’s approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = g\beta (T' - T_{\infty}') + g\beta' (C' - C_{\infty}') + \nu \frac{\partial^2 u'}{\partial z'^2}$$  \hspace{1cm} (1)

$$\frac{\partial v'}{\partial t'} + 2\Omega' u' = \nu \frac{\partial^2 v'}{\partial z'^2}$$  \hspace{1cm} (2)

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'}$$  \hspace{1cm} (3)

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z'^2}$$  \hspace{1cm} (4)

The term $\frac{\partial q_r}{\partial z'}$ represents the change in the radiative flux with distance normal to the plate with the following initial and boundary conditions:

$t' \leq 0$: $u' = 0$, $T' = T_{\infty}'$, $C' = C_{\infty}'$ for all $z'$

$t' > 0$: $u' = ct'$, $T' = T_w' + (T_w - T_{\infty}')At'$, $C' = C_w'$ at $z' = 0$ $u = 0$, $T \to T_w'$, $C' \to C_{\infty}'$ as $z' \to \infty.$

Here, $A = \frac{u_0^2}{v}$ by the Rosseland approximation [4], radiative heat flux of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial z'} = -4a^* \sigma(T'^4 - T''^4)$$  \hspace{1cm} (6)

It is assumed that the temperature differences within the flow are sufficiently small such that $T''^4$
may be expressed as a linear function of the temperature. This is accomplished by expanding $T'^4$ in a Taylor series about $T'_w$ and neglecting higher-order terms, thus

$$T'^4 \approx 4T'^3 - 3T'_w^4$$  \hspace{1cm} (7)

By using equations (6) and (7), equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y^2} + 16a^*\sigma T'_w (T'_w - T')$$

On introducing the following dimensionless quantities

$$(u,v) = \frac{(u',v')}{(v c)^{\frac{1}{3}}}, \quad t = t' \left(\frac{e^2}{\nu}\right)^{\frac{1}{3}}, \quad z = z' \left(\frac{c}{\nu^2}\right)^{\frac{1}{3}}, \quad \theta = \frac{T' - T'_w}{T'_w - T'}.$$  

$$Gr = \frac{g \beta(T'_w - T'_w)}{c}, \quad C = \frac{C' - C'_w}{C'_w - C'_w}, \quad Ge = \frac{g \beta^2(C'_w - C'_w)}{c}, \quad Pr = \frac{\mu C_p}{k}, \quad \Omega = \Omega' \left(\frac{v}{c^2}\right)^{\frac{1}{3}}, \quad R = \frac{16a^*\nu \sigma T'_w^3}{k} \left(\frac{v}{c^2}\right)^{\frac{1}{3}}.$$  

and the complex velocity $q = u + iv$, $i = \sqrt{-1}$ in equations (1) to (5), the equations relevant to the problem reduce to

$$\frac{\partial q}{\partial t} + 2i\Omega = Gr\theta + GeC + \frac{\partial^2 q}{\partial z^2},$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial z^2} - \frac{R}{Pr} \theta,$$  

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial z^2}$$  \hspace{1cm} (12)

The initial and boundary conditions in non-dimensional form are as follows:

$$q = 0, \quad \theta = 0, \quad C = 0, \quad \text{for all} \quad z \leq 0 \& t \leq 0$$

$$t > 0: \quad q = t, \quad \theta = t, \quad C = 1, \quad \text{at} \quad z = 0$$

$$q = 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0, \quad \text{as} \quad z \rightarrow \infty.$$  \hspace{1cm} (13)

All the physical variables are defined in the appendix.

3. Theoretical Results

3.1 Solution

The solutions are obtained for equations (10) to (12), subject to the boundary conditions (13), by the Laplace-transform technique and the solutions are derived as follows:

$$C = \text{erfc}(\eta \sqrt{Sc})$$  \hspace{1cm} (14)

$$\theta = \frac{t}{2} \left[\exp(2\eta \sqrt{Rt}) \text{erfc}(\eta \sqrt{Pr} + \sqrt{at}) + \exp(-2\eta \sqrt{Rt}) \text{erfc}(\eta \sqrt{Pr} - \sqrt{at})\right]$$

$$- \frac{\eta Pr \sqrt{t}}{2 \sqrt{R}} \left[\exp(-2\eta \sqrt{Rt}) \text{erfc}(\eta \sqrt{Pr} - \sqrt{at}) - \exp(2\eta \sqrt{Rt}) \text{erfc}(\eta \sqrt{Pr} + \sqrt{at})\right]$$  \hspace{1cm} (15)
\[ q = \frac{1}{2} \left( t + \frac{Gr}{b^2(1 - Pr)} + \frac{Gc}{c(1 - Sc)} + \frac{t Gr}{(1 - Pr)b} \right) \left[ \exp(2\eta \sqrt{mt}) \text{erfc}(d1) + \exp(-2\eta \sqrt{mt}) \text{erfc}(d2); \right. \\
\left. - \frac{\eta \sqrt{t}}{2\sqrt{m}} \left( \frac{Gr}{(1 - Pr)b} + 1 \right) \left[ \exp(-2\eta \sqrt{mt}) \text{erfc}(d2) - \exp(2\eta \sqrt{mt}) \text{erfc}(d1) \right] \right] \\
- \frac{Gr \exp(bt)}{2b^2(1 - Pr)} \left[ \exp(2\eta \sqrt{(b + m)t}) \text{erfc}(d3) + \exp(-2\eta \sqrt{(b + m)t}) \text{erfc}(d4) \right] \\
- \frac{Gc \exp(ct)}{2c(1 - Sc)} \left[ \exp(2\eta \sqrt{(c + m)t}) \text{erfc}(d5) + \exp(-2\eta \sqrt{(c + m)t}) \text{erfc}(d6) \right] \\
- \frac{Gr \exp(bt)}{2b^2(1 - Pr)} \left[ \exp(2\eta \sqrt{Pr(b + a)t}) \text{erfc}(d9) + \exp(-2\eta \sqrt{Pr(b + a)t}) \text{erfc}(d10) \right] \\
- \frac{1}{2} \left( \frac{Gr}{b^2(1 - Pr)} + \frac{t Gr}{b(1 - Pr)} \right) \left[ \exp(2\eta \sqrt{Rt}) \text{erfc}(d7) + \exp(-2\eta \sqrt{Rt}) \text{erfc}(d8) \right] \\
+ \frac{Gr \eta \sqrt{t} \sqrt{Pr}}{2(1 - Pr)b\sqrt{a}} \left[ \exp(2\eta \sqrt{Pr(b + a)t}) \text{erfc}(d7) - \exp(-2\eta \sqrt{Pr(b + a)t}) \text{erfc}(d8) \right] - \frac{Gc}{c(1 - Sc)} \text{erfc} \left( \eta \sqrt{Sc} \right) \\
+ \frac{Gc \exp(ct)}{2c(1 - Sc)} \left[ \exp(2\eta \sqrt{Sc \ ct}) \text{erfc}(d11) + \exp(-2\eta \sqrt{Sc \ ct}) \text{erfc}(d12) \right] \\
\]

where
\[ d1, d2 = [\eta \pm \sqrt{mt}] \]
\[ d3, d4 = [\eta \pm \sqrt{(b + m)t}] \]
\[ d5, d6 = [\eta \pm \sqrt{(c + m)t}] \]
\[ d7, d8 = [\eta \sqrt{Pr} \pm \sqrt{at}] \]
\[ d9, d10 = [\eta \sqrt{Pr} \pm \sqrt{(a + b)t}] \]
\[ d11, d12 = [\eta \sqrt{Sc} \pm \sqrt{ct}] \]
\[ \eta = \frac{z}{2\sqrt{t}}, \quad a = \frac{R}{Pr}, \quad b = \frac{R - m}{1 - Pr}, \quad c = \frac{m}{Sc - 1} \quad \text{and} \quad m = 2i\Omega. \]

In equation (16), the argument of the complementary error function and the error function is complex. Hence in order to obtain the \( u \) and \( v \) components of the velocity and skin-friction, we have used the following formula due to Abramowitz and Stegun [1]:
\[ \text{erf} (a + ib) = \text{erf} (a) + \frac{\exp(-a^2)}{2a\pi} \left[ 1 - \cos(2ab) + i \sin(2ab) \right] \\
+ \frac{2\exp(-a^2)}{\pi} \sum_{n} \exp(-n^2/4) \left[ f_n(a,b) + ig_n(a,b) \right] + e(a,b) \]

where
\[ f_n = 2a - 2a \cosh(nb) \cos(2ab) + n \sinh(nb) \sin(2ab) \]
\[ g_n = 2a \cosh(nb) \sin(2ab) + n \sinh(nb) \cos(2ab) \]
\[ |e(a,b)| \approx 10^{-16} |\text{erf} (a + ib)|. \]
3.2 Result Analysis

Using the above formula, expressions for $u$, $v$ are obtained, but they are omitted here to save the space. In order to get a physical view of the problem, these expressions are used to obtain the numerical values of $u$, $v$, for different values of the various parameters, such as rotation, radiation, Schmidt number, thermal Grashof number, and mass Grashof number.

Figure 1 depicts the concentration profiles for different values of the Schmidt number ($Sc = 0.16, 0.24, 0.6, 0.78$) at time $t = 0.2$. Wall concentration increases with decreasing Schmidt number. It is observed that there is a fall in concentration due to increasing values of the Schmidt number.

The temperature profiles for air ($Pr = 0.71$) are calculated for different values of thermal radiation parameter from Equation (15) and these are shown in Fig. 2. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter as well as the time.

The primary velocity profiles of air for different values of the radiation parameter...
(R=2, 50), Gr = 5, Gc = 5, Sc = 0.6, t = 0.2, Pr = 0.71 and rotation parameter (Ω = 0.5, 2, 3) are shown in Fig. 3. It is observed that the primary velocity increases with decreasing radiation parameter R as well as the rotation parameter Ω in cooling of the plate. This shows that primary velocity decreases in the presence of high thermal radiation and rotation.

The secondary velocity profiles of air for different values of the radiation parameter (R = 2, 50), Gr = 5, Gc = 5, Sc = 0.6, t = 0.2, Pr = 0.71 and rotation parameter (Ω = 0.5, 2, 3) are shown in Fig. 4, the effect of radiation increases the secondary velocity v. But the effect of rotation on v is just reverse to that of radiation parameter.

The primary velocity profiles for different thermal Grashof number (Gr = 3, 5), mass Grashof number (Gc = 3, 5), Sc = 0.6, and time t = 0.2 are shown in Fig. 5. It is clear that the primary velocity increases with increasing thermal Grashof number or mass Grashof number.

Figure 6 depicts the secondary velocity profiles for different thermal Grashof number (Gr = 3, 5), mass Grashof number (Gc = 3, 5), Sc = 0.6, and time t = 0.2. It is observed that the secondary velocity decreases with increasing thermal Grashof number or mass Grashof number.

4. Conclusions

Theoretical analysis is performed to study flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of thermal radiation in a rotating fluid. The dimensionless governing equations are solved by Laplace-transform technique. The conclusions of the study are as follows:

- Concentration falls with the raise in Schmidt number.
- Temperature is enhanced with the decreasing radiation parameter and increasing time.
- The influence of the radiation or rotation parameter on the primary flow has a retarding effect for cooling the plate. This phenomenon helps in designing equipments.
- The secondary velocity is enhanced with the raise in thermal radiation and opposite phenomenon occurs with the rotation parameter.
- Primary velocity is enhanced with an increase in thermal Grashof number or mass Grashof number, but the secondary velocity has a reverse phenomenon.
5. Appendix: Notation

- $a'$: absorption coefficient [m$^{-1}$]
- $C'$: concentration [kgm$^{-3}$]
- $C$: dimensionless concentration
- $C_p$: specific heat at constant pressure [Jkg$^{-1}$K$^{-1}$]
- $D$: mass diffusion coefficient [m$^2$s$^{-1}$]
- $g$: acceleration due to gravity [ms$^{-2}$]
- $Gr$: thermal Grashof number [-]
- $Gc$: mass Grashof number [-]
- $k$: thermal conductivity of the fluid [Wm$^{-1}$K$^{-1}$]
- $Pr$: Prandtl number [-]
- $q_r$: radiative heat flux in the y-direction [Wm$^{-2}$]
- $R$: radiation parameter [-]
- $Sc$: Schmidt number [-]
- $T'_w$: temperature of the fluid far away from the plate [-]
- $T'_t$: temperature of the plate [K]
- $T'$: temperature of the fluid near the plate [-]
- $t'$: time [s]
- $t$: dimensionless time
- $u'$: velocity of the fluid in the $x'$-direction [ms$^{-1}$]
- $u$: dimensionless velocity
- $v'$: velocity of the fluid in the $y'$-direction [ms$^{-1}$]
- $v$: dimensionless velocity
- $y'$: coordinate axis normal to $x'$-axis [-]
- $z'$: coordinate axis normal to the plate [-]
- $z$: dimensionless coordinate axis normal to the plate

**Greek Symbols**

- $\beta$: volumetric coefficient of thermal expansion [K$^{-1}$]
- $\beta'$: volumetric coefficient of expansion with concentration [K$^{-1}$]
- $\mu$: coefficient of viscosity [Pa.s]
- $\nu$: kinematic viscosity [m$^2$s$^{-1}$]
- $\Omega'$: rotation parameter [rad]
- $\Omega$: dimensionless rotation parameter
- $\rho$: density [kg.m$^{-3}$]
- $\tau$: dimensionless skin-friction [kgm$^{-1}$s$^{-2}$]
- $\sigma$: Stefan-Boltzmann constant [Wm$^{-2}$K$^{-4}$]
- $\theta$: dimensionless temperature
- $erfc$: complementary error function

**Subscripts**

- $w$: conditions on the wall
- $\infty$: free stream conditions
References


